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ABSTRACT

Fluidelastic instability (FEI) in tube arrays has been studied extensively experimentally and theoretically for the last 50 years, due to its potential to cause significant damage in short periods of time. Incidents similar to those observed at SONGS indicate that the problem is not yet fully understood, probably due to the large number of factors affecting the phenomenon. In this study, a new approach for the analysis and interpretation of FEI data using machine learning algorithms is explored. FEI data for both single and two-phase flows has been collected from the literature and utilized for training a machine learning algorithm in order to either provide estimates of the reduced velocity (single and two-phase), or indicate if the bundle is stable or unstable under certain conditions (two-phase). The analysis included the use of logistic regression as a classification algorithm for two-phase flow problems to determine if specific conditions produce a stable or unstable response. The results of this study provide some insight into the capability and potential of logistic regression models to analyze FEI if appropriate quantities of experimental data are available.

1 Introduction

Fluidelastic Instability (FEI) in tube arrays is a flow-induced vibration mechanism with potential to cause significant structural damage of tube arrays in a short period of time. Prediction and avoidance of FEI are a key design objective for equipment operating in the thermal, nuclear and power industries in order to eliminate the need for unscheduled stops and unnecessary maintenance. Extensive studies have been carried out to improve the physical understanding of FEI and identify the parameters that influence the onset of instability in both single and two-phase flows, usually focusing on occurrence of FEI in the transverse direction. The mechanism was first introduced by Roberts [1] and a design framework was proposed by Connors [2] by studying force and tube displacement in a single row of cylinders. Connors quasi-static model provided a relationship aimed to predicting FEI based on the two dimensionless parameters (reduced velocity and mass-damping parameter) introduced by Roberts and based on both flow and structural variables:

$$\frac{U_{cr}}{fD} = K\left(\frac{m\delta}{\rho D^2}\right), \tag{1}$$

where U_{cr} is the flow critical velocity at instability, f is the tube frequency, D is the tube diameter, ρ is the fluid density and δ is the damping. Further research carried out during the following two decades provided valuable insight into the physics of FEI with the development of semi-empirical models (Lever and Weaver [3], Yetisir and Weaver [4]), unsteady models (Tanaka and Takahara [5]), quasi-steady models (Price and Païdoussis [6]) and quasi-unsteady models (Granger and Païdoussis [7]). However, these models required additional information for prediction of FEI, including force coefficients, time delays and turbulence. Significant scatter may be observed in FEI results from different researchers, possibly caused by parameter definition differences among existing studies [8].

For two-phase flows, studies on FEI include experiments in steam-water mixtures [9,10], air-water [11,12] and refrigerants [13]. Nuclear steam generators, which require operation that avoids FEI for safety reasons, are subjected to steam-water flows, which makes two-phase one-component mixtures particularly suitable in terms of more accurately simulating phase change phenomena, mass ratio and density ratio. However, consideration of two phases requires assumptions to determine how to calculate the flow velocity and density and introduces some difficulty estimating the damping required by equation 1.

More recently, FEI in the streamwise direction has become an important subject of study due to the failure of steam generators at the San Onofre Nuclear Generating Station (SONGS), which have been attributed to the occurrence of this phenomenon [14,15].

In order to reduce some of the challenges presented by experimental studies, numerical modelling of FEI has been gaining traction in the last decade. Selima et al. [16] utilized an analytical frame work to analyze FEI in two-phase flows. Sadek et al. [17] implemented Computational Fluid Dynamics (CFD) to analyze the flow field and compute fluid and structural forces in order to predict the onset of instability.

Currently, determination of the critical velocity for onset of FEI at the design stage is typically carried out by using empirical stability maps [18–21], both for single and two-phase flows. These maps, based on reduced velocity and mass-damping parameter, do not take into account the type of fluid, tube materials, P/D ratio, phase change, density ratio or liquid and gas viscosities.

Machine Learning (ML), a subset of Artificial Intelligence algorithms, is especially useful to analyze large quantities of data where the desired outcome (or target) is affected by many variables. For the case of FEI, extensive modelling has demonstrated the influence of several parameters and designing experiments (either numerical or practical) that accommodate variation of all of them is not possible, nor is desirable. ML algorithms can find trends and patterns while filtering data through 'layers' in order to discover how different parameters interact and which ones are relevant to obtain a specific outcome. If enough data is available to 'train' the ML algorithm, the latter will be capable of predicting an outcome independently of the number of variables involved or how these variables are related to each other. ML has been used in vibration control by Yang et al. [22], who utilized supervised ML algorithms coupled with computer vision to estimate the tension in a vibrating cable by analyzing its motion pattern, in contract to the existing methods that use load cells and accelerometers. Ren et al. [23] implemented an active control mechanism based on CFD and ML to avoid vortex-induced vibrations in a single cylinder, achieving reductions of 90% and 25% in amplitude and drag respectively.

The objective of this research is to present a new approach for the analysis and of FEI data using machine learning algorithms. Potential contributions include better control of the dynamic response of the array during equipment operations by adjusting input heat and/or flow rate and the possibility of designing practical experiments based on parameters that are relevant for a specific case. To this end, FEI data for both single and two-phase flows has been collected from the literature and utilized for training a ML algorithm, in order to analyze the relative importance of the parameters involved and to provide an estimate of the reduced velocity that can be translated into a stability map.

2 Methodology

Three groups of data (or datasets) were defined based on FEI results available in the literature. The first and second groups corresponded to fluidelastic data for single and two-phase flows respectively, extracted from the references in Table 1. For these groups, the dataset includes only points where FEI has already occurred, and consequently, they all represent conditions for which the tube bundle has become unstable. The data, extracted from stability maps, includes critical velocity and mass-damping parameter values (based on certain assumptions in the case of two-phase flow). For the most part, information regarding the type of fluid, bundle geometry, tube mass and P/D ratio is also provided. However, additional details (fluid properties, damping, temperature, pressure, etc.) are not commonly reported. In this study, the analysis has been focused on parallel triangular tube arrays since they are more susceptible to experience FEI and therefore there is more experimental data available.

The third dataset consists of two-phase flow experiments where all details of the measurements, calculations, fluid properties and assumptions are known. These included results reported by Moran [37]. For these cases, it is known if the tube array is stable or unstable for each experimental trial, in addition to the corresponding FEI threshold. They also provide values of many of the parameters involved (temperature, surface tension, viscosity, density, etc.) for each of the stable/unstable bundle conditions. In order to provide a common frame of comparison for the data available, fluid properties and void fraction calculated using Homogeneous Equilibrium Model (HEM) will be considered. It should be noted that, given the appropriate amount and categorization of data, the analysis can be extended to cover any given range of properties available from experimental and/or numerical results.

Source	Fluid	Case
Hartlen (1974) [24]	Air	SP
Gorman (1976) [25]	Water	SP
Weaver and Gorman (1976) [26]	Air	SP
Pettigrew et al (1978) [27]	Water	SP
Chen and Jendrzejczyk (1978) [28]	Water	SP
Heilker and Vincent (1981) [29]	Water	SP
Weaver and El-Kashlan [30]	Air	SP
Axisa et al. (1985) [31]	Steam-Water	TP
Pettigrew et al. (1989) [32]	Air-Water	TP
Pettigrew et al. (1995) [33]	R-22	TP
Mann and Mayinger (1995) [34]	R-12	TP
Feenstra et al. (1995) [13]	R-11	TP
Feenstra et al. (2002) [35]	R-11	TP
Moran and Weaver (2019) [36]	R-11	TP

Table 1. Research studies used for training and testing of machine learning algorithms, single phase (SP) and two-phase (TP) flow.

For the analysis, two machine learning approaches were used due to the size of the datasets available: linear regression and logistic regression, a classification learning algorithm. The linear regression is a learning algorithm that produces a model resulting from linear combination of the input parameters (or dimensions). In a collection of labelled samples $(x_i, y_i)_{i=1}^N$, where N is the size of the collection, x_i is the D-dimensional vector of samples $i = 1, ..., N$ and y_i is the target value, we want to construct a model $f_{w,b}(x)$ as a linear combination of the samples in x such as:

$$f_{w,b}(x) = wx + b, \tag{2}$$

where w is an input vector of parameters and b is a real number. The model will be implemented in way that given a value of x, a corresponding $y = f_{w,b}(x)$ can be predicted. Linear models are simple and can indicate the influence (or weights) of the inputs and how they affect output values. These models can outperform more complex nonlinear algorithms in situations with small numbers of training datasets or sparse data [38].

For FEI analysis, the problem can also be considered as one of binary classification where it is required to differentiate or separate 'stable' and 'unstable' conditions. In this case, a logistic regression can be used. This method is fundamentally different than a linear regression, since the latter is not a classification algorithm. In addition, the logistic approach utilizes a non-linear function to produce an output. The logistic regression method assumes y_i will still be modelled as a function of x_i, but results will be classified into two possible groups (stable and unstable in this study) based on the input vector. Using the approach suggested by equation 2 may not be practical, since this form of linear combination is a function that can attain any value between $-\infty$ and ∞, but y_i should have only two possible outputs. The logistic approach suggests that negative and positive labels are defined (zero and one respectively) in order to find a continuous function whose codomain is (0, 1). Once this is accomplished, if the value returned by the model for input x is closer to 0, a negative label can be assigned to x. If not, the example is labeled as positive. The standard logistic function used in this study (sigmoid):

$$f_{w,b}(x) \stackrel{\text{def}}{=} \frac{1}{1 + e^{-wx+b}}. \tag{3}$$

The logistic approach will use a maximum likelihood optimization criterion to provide solutions for equation 3, instead of minimizing the loss function as in linear regression. The maximum likelihood of the training data is given by:

$$L_{w,b} \stackrel{\text{def}}{=} \prod_{i=1...N} f_{w,b}(x_i)^{y_i}(1-f_{w,b}(x_i))^{(1-y_i)} \quad (4)$$

This study will use the gradient descent method to solve equation 4, which is clearly not linear in nature.

3 ML Modelling

3.1 Single-phase model

The purpose of using the linear regression approach to model single-phase data was, in principle, to test the general ability of ML to model FEI. To this end, the data was collected for all four bundle geometric configurations, different fluids and P/D ratios, in order to have large enough dataset to train the model. The latter parameter was not included in the algorithm, as the authors wanted first to evaluate if the model could replicate the trend observed in the stability maps. The fluid type was assumed to be 1 for air and 2 for water, based on information available in Table **??**. The input vector had two parameters in addition to fluid type: tube bundle configuration and mass damping parameter ($D = 3$). The only output of the model was reduced flow velocity. The dataset corresponding to single-phase FEI described in the methodology section was divided randomly into a training set (70% of the data available) and a test set (the remaining 30%). The input data were normalized between 0 to 1 with respect to the mean and standard deviation values for each variable within the dataset. This operation allows the model to select appropriate weights or 'contributions' for each parameter that will be expressed quantitatively between 0 and 1. A regulation parameter was used to avoid overfitting of the ML algorithm to the training set. Several values for the regulation parameter were tested for the purpose of optimizing the model in terms of minimizing the error between the model results and the test dataset.

3.2 Two-phase model: linear regression

Based on FEI data available, the input vector for the linear regression model was different than in the single-phase case, to include: fluid type, P/D ratio, tube diameter, frequency in air, tube mass per unit length, liquid density, gas density, liquid viscosity, surface tension, damping ratio in air and HEM void fraction ($D = 11$). One of the objectives of this study was to explore the effect of variables that are not taken into consideration in Connors equation, and investigate their influence on the FEI threshold. Only experimental data corresponding to the parallel triangle configuration was used for this model. Similarly to the single-phase model, the dataset was divided randomly into training and test sets (70% and 30% respectively). The input data was also normalized between 0 to 1 in order to let the model assign the proper weights for different parameters.

3.3 Two-phase model: logistic regression

The objective of using the logistic regression in combination with the two-phase flow data is to try to predict if for a given set of parameter combinations, the tube bundle will be stable or unstable, instead of providing a numerical estimate of the reduced velocity. The authors believe that ML can help prevent the occurrence of FEI during equipment operation. Experimental data was collected from Moran [37], since it contained the values of all measured and calculated flow parameters during the experiments to find the critical velocity, and therefore offer information on stable and unstable conditions for training the ML model. The limitation of this approach is that since all experimental values provided by Moran [37] come from the same experimental rig, tube bundle and working fluid (R-11), the logistic model was unable to study the effect of fluid type, damping and P/D ratio. However, the methodology used in this study will improve its prediction capabilities if more stable/unstable data is available for training, including experimental and numerical sources. The input vector for the logistic model includes: temperature, frequency in air, void fraction, pitch velocity, liquid viscosity, gas viscosity, liquid density, gas density, liquid enthalpy, latent heat, saturation pressure and surface tension ($D = 12$). The dataset was divided randomly into training set (70%) and test set (30%) and a regulation parameter was used to avoid overfitting the model to the training set.

4 Results

4.1 Single-phase flow FEI Prediction

As mentioned in the section above, the purpose of testing the ML algorithm using single-phase data was to examine its general potential to predict FEI. Although single-phase physical models have shown ability to predict the instability threshold relatively accurately, this step was important in the current research to validate the feasibility of Machine Learning. Figure 1 shows the predicted reduced flow velocity plotted against the testing dataset on a linear scale. The ML prediction shows good agreement with the experimental data for values of U_r lower than 30, at which point the correlation seems to change. This is

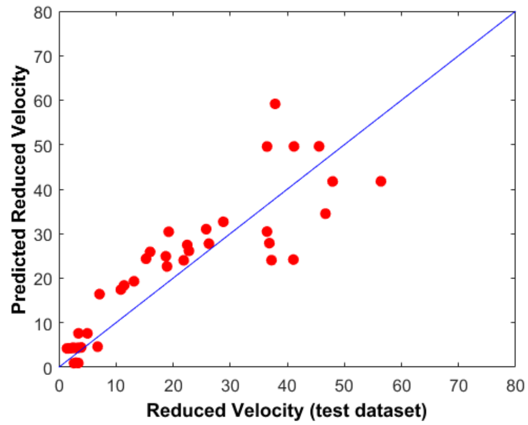

Fig. 1. Correlation between predicted reduced velocity and test sub-dataset for single phase flow.

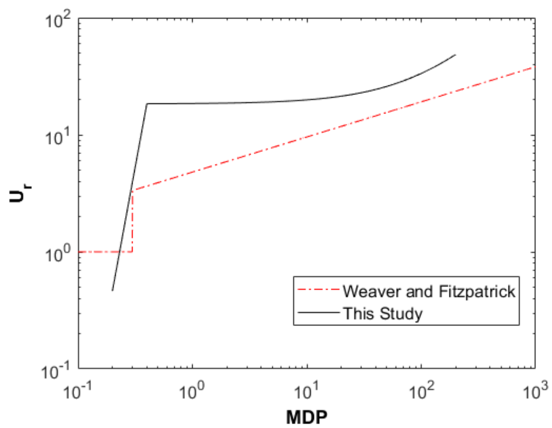

Fig. 2. Stability map generated for parallel triangular arrays based on single-phase training dataset.

due to the scatter and scarcity of the data present in the literature available for training the ML algorithm. The corresponding value of the coefficient of determination R^2 in Figure 1 is 0.79, indicating an acceptable correlation between the testing and predicted data series. At this point, the ML model shows encouraging results given the limitation of the training data available.

Figure 2 presents the predictions of the ML model plotted on the stability map by Weaver and Fitzpatrick [18] for the parallel triangle bundle geometry. It can be observed that the model suggests a less conservative boundary than the guidelines proposed by Weaver and Fitzpatrick since it was trained with the experimental data, while the limit shown in the stability maps was defined as a lower bound to the FEI literature available at the time. However, for values of the mass-damping parameter between 0.3 and 3, the ML model predicts a reduced velocity significantly larger than expected. The deviation between the prediction of the algorithm and the bound indicated by Weaver and Fitzpatrick may be due to the lack of experimental data for low values of the mass-damping parameter. However, the model captures the discontinuity of the FEI critical velocity at $\frac{m\delta}{\rho D^2} = 0.25$. The authors are currently in the process of gathering more experimental data, which will significantly improve the output of the machine learning algorithm.

4.2 Two-phase flow: Linear Regression

Following the methodology described in section 3.2, the FEI data for two–phase flow experiments in parallel triangular tube bundles was used to train the ML algorithm. The data comprises different fluids and combination of other parameters, and the ML was used to estimate the relative importance of each in the FEI threshold. Figure 3 shows the weights or contributions of each parameter according to the ML algorithm.

The larger contributors to the FEI threshold seem to be frequency, diameter, liquid density and liquid viscosity. The first two parameters are part of the reduced velocity, and liquid density is included in the mass-damping parameter. The relatively small influence of damping in this figure is explained by the use of structural damping in the model, which represents a small portion of the energy dissipation capability of the two-phase flow. The two–phase damping was internally estimated

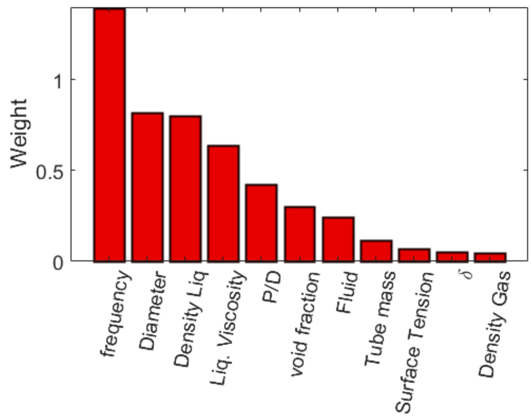

Fig. 3. Weights assigned to each parameter in the input vector by the ML algorithm, linear regression in two-phase flow.

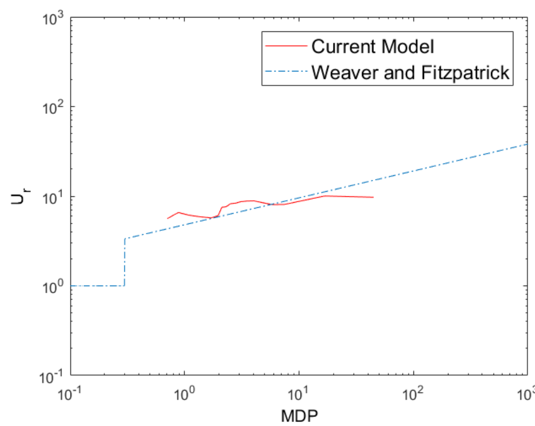

Fig. 4. Stability map generated from the logistic regression results for the data of [37], two-phase flow in parallel-triangular array.

by the model and used for the prediction of the FEI threshold. Friction and squeeze film damping at the supports were not considered in this implementation of the model. The liquid viscosity represents a form of energy dissipation and has an important role in flow configuration and distribution within the tube bundle. Void fraction, which has a significant influence on the tube dynamic response in two-phase flow, also represents an important contribution to the FEI threshold. It is important to note that the dataset used to generate this result contains only the conditions at which the tube becomes unstable for each experiment.

4.3 Two-phase flow: Logistic Regression

Figure 4 shows a comparison of the logistic regression results with the stability map proposed by Weaver and Fitzpatrick [18]. In this case, the logistic model has been trained with a complete set of experimental data from Moran [37], including flow conditions for which the tube bundle was either stable and unstable. In figure 4, the red line indicates an 'unstable' tube array envelope prediction from the logistic regression model. It can be seen that the model agrees well with the stability threshold suggested by Weaver and Fitzpatrick, albeit the prediction is done for the range of data gathered by Moran [37]. As described in section 3.3, the flow temperature, which affects the fluid properties significantly, was included in this analysis. The results shown in 4 present some scatter that might be due to the fact that temperature changed for different void fractions of the one-component two-phase mixture, affecting surface tension, viscosity and density (influencing flow regime). Further investigation of the phenomenon is required to confirm this hypothesis.

5 Conclusions

A new approach for analyzing fluidelastic instability using machine learning algorithms has been presented. A linear regression model was used for single and two-phase fluidelastic data available in the literature for different fluids and array configurations, while logistic regression was used to analyze a specific set of two-phase flow experiments. The results obtained in single and two-phase flows show a qualitative agreement with trends observed in experiments, but more data in terms of both quantity and detail is required to better train and test the algorithms for use in equipment control and avoidance of conditions that can lead to FEI. The use of statistical learning techniques also proved useful to identify the influence of several structural and/or flow parameters on the occurrence of FEI, and can potentially be used for designing more efficient experiments to enhance our understanding of the physical mechanisms that produce fluidelastic instability.

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