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Article

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Abstract: The primary objective of the paper is to present the Banach contraction theorem in soft fuzzy metric spaces while taking into consideration a restriction on the soft fuzzy metric between the soft points of the absolute soft set. A new altering distance function, namely the Ψ-contraction function, is introduced on soft fuzzy metric spaces, and some fixed point results are proven by considering soft mappings that comprise Ψ-contraction with the continuity of soft t-norm. In addition to that, some illustrations are supplied for the support of the established soft fuzzy Banach contraction theorem and fixed point results over Ψ-contraction mappings. The obtained results generalize and extend some well-known results present in the literature on fixed point theory.

Keywords: fixed point; soft metric space; soft fuzzy metric spaces; altering distance functions; contraction mapping

MSC: 47H10; 54H25

1. Introduction

The problems that arise in everyday life include uncertain information that cannot be adequately expressed in conventional mathematics. Fuzzy set theory, developed by Zadeh [1], and the theory of soft sets, introduced by Molodstov [2], are two distinct kinds of mathematical concepts capable of being utilized when dealing with uncertainties. Both of these techniques have their advantages in addressing issues across all domains. Functional analysis studies had been advanced by Banach's formulation of the renowned Banach Contraction Principle [3]. The Banach contraction principle is one of the most important results of fixed point theory which has undergone intensive research. The study's objective is to put forth an unfamiliar contraction mapping principle in soft fuzzy metric spaces which is soft generalization of fuzzy metric spaces.

The theory of soft sets was first initialized through Molodtsov [2] as an elementary mathematical mode to tackle the ambiguities in data. Maji et al. [4,5] executed a study over soft sets that have made progress in the soft set theory. They analyzed the soft set study and presented an implementation of the same to decision-making situations. Many authors maintained the study of the soft set theory and its applications across different disciplines after that [2,5–9]. Similar work can be seen in the context of rough sets and extensions [10]. Through the introduction of the concepts of soft metric space based on soft points in soft sets, Das and Samanta [11–13] provided a great contribution to this area. On the other hand, L.A. Zadeh in 1965 delivered a remarkable idea of fuzzy sets, and since then it has evolved into a crucial mechanism for resolving cases involving ambiguity and uncertainty [1]. To produce Hausdorff topology to a certain category of fuzzy metric spaces, Kramosil and Michalek's definition of a fuzzy metric space was modified by George and Veeramani [14,15]. By merging the idea of two or more generalizations of a metric space, researchers obtained strong results related to fixed points and presented their findings [16–20]. Various results on fixed points have been developed in several generalizations of metric spaces, which implement the idea of altering distance, as referenced in articles [21–24].

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By fusing the conceptions of soft metric spaces and fuzzy metric spaces, Beaula and Raja [25] created the idea of a fuzzy soft metric space and developed several concepts that utilize the fundamental knowledge of fuzzy soft sets. Later, more investigations were conducted in fuzzy soft metric spaces [26,27]. In 2017, Ferhan Sola Erduran [8] proposed the idea of a soft fuzzy metric with fundamental characteristics and topological structure in soft fuzzy metric spaces by applying the concepts of soft points and soft real numbers. The study was further explored by introducing concepts such as countability, convergence, and completeness in soft fuzzy metric spaces, compact soft fuzzy metric spaces, and totally fuzzy bounded spaces [6,28–30]. Any version of the Banach contraction principle has not been proven in a soft fuzzy metric space. To cover this research gap and study soft fuzzy metric spaces, we introduced the soft fuzzy contraction and a new kind of altering distance function, namely the Ψ-function with the establishment of several fixed point results in soft fuzzy metric spaces using the soft fuzzy contraction mapping and the Ψ-contraction mapping.

The following describes the structure of the paper. Some characteristics and fundamental ideas of soft fuzzy metric spaces are provided in Section 2. The concept of contraction in soft fuzzy metric spaces, Ψ-function, Ψ-contraction mapping in soft fuzzy metric spaces, followed by the soft fuzzy contraction theorem and fixed point results for Ψ-contraction mappings are all introduced in Section 3. The established results are further supported by some examples. Section 4 of this work contains its conclusion.

2. Preliminaries

In this section, we provide some fundamental definitions for establishing the main results. The universal set, the assembly of parameters, and the collection of all subsets of K are indicated with the notations $K, \mathcal{P}$, and $P(K)$, respectively.

For more information, we recommend [2,5,6,8,11,13,30,31], etc.

Definition 1. *A pair $(\mathcal{W}, \mathcal{P})$ is called a soft set over the universal set K if $\mathcal{W}$ is a function from the parameter set $\mathcal{P}$ to the power set of K, i.e., $\mathcal{W} : \mathcal{P} \to P(K)$ [2] .*

Definition 2. *A soft set $(\mathcal{W}, \mathcal{P})$ over K is called an absolute soft set if $\mathcal{W}(\eta) = K$ for all $\eta \in \mathcal{P}$. $\tilde{\mathcal{K}}_{\mathcal{P}}$ shall be used to represent the absolute soft set over K with parameter set $\mathcal{P}$ [5].*

Definition 3. *A soft set $(\mathcal{W}, \mathcal{P})$ over the universal set K is called a null or void soft set if $\mathcal{W}(\eta) = \{\}$ for all $\eta \in \mathcal{P}$. This is noted by $\tilde{\phi}$ [11].*

We consider $\mathbb{R}$ as the collection of all real numbers. We signify the assembly of all non-void bounded subsets of $\mathbb{R}$ with $\mathbf{B}(\mathbb{R})$.

Definition 4. *A pair $(\mathcal{W}, \mathcal{P})$ is called a soft real set if $\mathcal{W}\colon \mathcal{P} \to \mathbf{B}(\mathbb{R})$. A soft real set $(\mathcal{W}, \mathcal{P})$ is called a soft real number if, for each $\eta \in \mathcal{P}$, $\mathcal{W}(\eta)$ is a singleton member of $\mathbf{B}(\mathbb{R})$. It is signified by $\tilde{x}$.*

For a soft real number $\tilde{x}$, if $\tilde{x}(\eta) = \{k\}$ for some $k \in \mathbb{R}$, then we denote it by $\bar{k}$ [11].

Definition 5. *A soft set over the universal set K is said to be a soft point if for exactly one parameter $\eta \in \mathcal{P}$, $\mathcal{W}(\eta) = \{x\}$ where $x \in K$ and $\mathcal{W}(\mu) = \phi$ for all $\mu \in \mathcal{P} \setminus \{\eta\}$. It is signified by $\tilde{x}_\eta$.*

A soft point $\tilde{x}_\eta$ is said to belong to a soft set $(\mathcal{W}, \mathcal{P})$ if $\tilde{x}_\eta = \{x\} \subseteq \mathcal{W}(\eta)$. This is also written as $\tilde{x}_\eta \tilde{\in} (\mathcal{W}, \mathcal{P})$. The collection of all soft points of $(\mathcal{W}, \mathcal{P})$ is signified with $SP(\mathcal{W}, \mathcal{P})$ [12].

Definition 6. *Any function from a parameter set $\mathcal{P}$ to the universal set K is called a soft element. In other words, a soft element is a function $\theta : \mathcal{P} \to K$. The soft set developed from grouping $\mathcal{M}$ of soft elements is denoted by $SS(\mathcal{M})$ [12].*

The assembly of all soft real numbers and non-negative soft real numbers with a parameter set $\mathcal{P}$ is denoted by $\mathbb{R}(\mathcal{P})$ and $\mathbb{R}(\mathcal{P})^*$, respectively. The collection of all soft real numbers in the intervals $[a, b]$ and $[0, \infty)$ is signified as $[a, b](\mathcal{P})$ and $[0, \infty)(\mathcal{P})$, respectively.

Definition 7. *For two soft real numbers $\tilde{w}$ and $\tilde{v}$, the following operations are defined [11]:*

$(\tilde{w} \oplus \tilde{v})(\eta) = \{\tilde{w}(\eta) + \tilde{v}(\eta), \eta \in \mathcal{P}\}$,
$(\tilde{w} \ominus \tilde{v})(\eta) = \{\tilde{w}(\eta) - \tilde{v}(\eta), \eta \in \mathcal{P}\}$,
$(\tilde{w} \circ \tilde{v})(\eta) = \{\tilde{w}(\eta) \,.\, \tilde{v}(\eta), \eta \in \mathcal{P}\}$.

Definition 8. *We consider $\tilde{\mathcal{K}}_{\mathcal{P}}$ as an absolute-soft set on a universal set. A mapping $\mathfrak{L}\colon SP(\tilde{\mathcal{K}}_{\mathcal{P}}) \times SP(\tilde{\mathcal{K}}_{\mathcal{P}}) \to \mathbb{R}(\mathcal{P})^*$ is claimed to be a soft metric over $\tilde{\mathcal{K}}_{\mathcal{P}}$ if the below-stated conditions are true [13]:*

(SM1) $\mathfrak{L}(\tilde{r}_{p_i}, \tilde{s}_{p_j}) \tilde{\geq} \bar{0}$ *for all* $\tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$,
(SM2) $\mathfrak{L}(\tilde{r}_{p_i}, \tilde{s}_{p_j}) = \bar{0} \iff \tilde{r}_{p_i} = \tilde{s}_{p_j}$,
(SM3) $\mathfrak{L}(\tilde{r}_{p_i}, \tilde{s}_{p_j}) = \mathfrak{L}(\tilde{s}_{p_j}, \tilde{r}_{p_i})$ *for all* $\tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$,
(SM4) $\mathfrak{L}(\tilde{r}_{p_i}, \tilde{t}_{p_k}) \tilde{\leq} \mathfrak{L}(\tilde{r}_{p_i}, \tilde{s}_{p_j}) + \mathfrak{L}(\tilde{s}_{p_j}, \tilde{t}_{p_k})$ *for each* $\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{t}_{p_k} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$.

The soft metric $\mathfrak{L}$ together with the absolute-soft set $\tilde{\mathcal{K}}$ is called a soft metric space. It is denoted as $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{L})$ or $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{L}, \mathcal{P})$ and abbreviated as SMS.

Definition 9. *We consider two soft metric spaces $(\tilde{\mathcal{K}}_{\mathcal{P}_{\mathcal{A}}}, \mathfrak{L}, \mathcal{P}_{\mathcal{A}})$ and $(\tilde{\mathcal{M}}_{\mathcal{P}_{\mathcal{B}}}, \mathfrak{L}, \mathcal{P}_{\mathcal{B}})$. Also, we consider function $(\mathcal{U}, \Phi) : (\tilde{\mathcal{K}}_{\mathcal{P}_{\mathcal{A}}}, \mathfrak{L}, \mathcal{P}_{\mathcal{A}}) \to (\tilde{\mathcal{M}}_{\mathcal{P}_{\mathcal{B}}}, \mathfrak{L}, \mathcal{P}_{\mathcal{B}})$. Then, $(\mathcal{U}, \Phi)$ is a soft mapping if $\mathcal{U}\colon \tilde{\mathcal{K}}_{\mathcal{P}_{\mathcal{A}}} \to \tilde{\mathcal{M}}_{\mathcal{P}_{\mathcal{B}}}$ and $\Phi\colon \mathcal{P}_{\mathcal{A}} \to \mathcal{P}_{\mathcal{B}}$ [31].*

Definition 10. *The collection of ordered pairs, $\mathcal{S}_F = \{(\tilde{x}_{p_i}, \mu_{\mathcal{S}_F}(\tilde{x}_{p_i})) | \tilde{x}_{p_i} \in \tilde{\mathcal{K}}_{\mathcal{P}}, p_i \in \mathcal{P}\}$, is a soft fuzzy set in $\tilde{\mathcal{K}}_{\mathcal{P}}$ wherein $\mu_{\mathcal{S}_F}$ is called a soft membership function which is a map from $\tilde{\mathcal{K}}_{\mathcal{P}}$ to $[0,1](\mathcal{P})$. Here, $\mu_{\mathcal{S}_F}(\tilde{x}_{p_i})$ represents the associated soft membership grade of soft point $\tilde{x}_{p_i}$ in $\mathcal{S}_F$ [8].*

Definition 11. *We consider function $\tilde{\circledast} : [0,1](\mathcal{P}) \times [0,1](\mathcal{P}) \longrightarrow [0,1](\mathcal{P})$; then, $\tilde{\circledast}$ is purported as a continuous soft t-norm if $\tilde{\circledast}$ agrees with the below-listed conditions [8]:*

(i) $\tilde{\circledast}$ *follows commutativity and associativity laws;*
(ii) *continuity of* $\tilde{\circledast}$,
(iii) $\tilde{c} \tilde{\circledast} \bar{1} = \tilde{c}$ *for all* $\tilde{c} \in [0,1](\mathcal{P})$,
(iv) $\tilde{c} \tilde{\circledast} \tilde{d} \leq \tilde{p} \tilde{\circledast} \tilde{q}$ *whenever* $\tilde{c} \leq \tilde{d}$ *and* $\tilde{p} \leq \tilde{q}$ *for* $\tilde{c}, \tilde{d}, \tilde{p}, \tilde{q} \in [0,1](\mathcal{P})$.

Example 1. $\tilde{\lambda} \tilde{\circledast} \tilde{\mu} = min\{\tilde{\lambda}, \tilde{\mu}\}$, $\tilde{\lambda} \tilde{\circledast} \tilde{\mu} = max\{\bar{0}, \tilde{\lambda} \oplus \tilde{\mu} \ominus \bar{1}\}$, *and* $\tilde{\lambda} \tilde{\circledast} \tilde{\mu} = \tilde{\lambda} \circ \tilde{\mu}$.

Definition 12. *We assume $\mathfrak{S}_F$ as a mapping $\mathfrak{S}_F : \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times (0,\infty)(\mathcal{P}) \longrightarrow [0,1](\mathcal{P})$. Then, $\mathfrak{S}_F$ is purported to be a soft fuzzy metric (abbreviated as SFM) on $\tilde{\mathcal{K}}_{\mathcal{P}}$ if [8]*

(SfM1) $\mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) \tilde{\geq} \bar{0}$ *for all* $\tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$, $\tilde{k} \tilde{>} \bar{0}$,
(SfM2) $\mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) = \bar{1} \iff \tilde{r}_{p_i} = \tilde{s}_{p_j}$ *for all* $\tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$, $\tilde{k} \tilde{>} \bar{0}$,
(SfM3) $\mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) = \mathfrak{S}_F(\tilde{s}_{p_i}, \tilde{r}_{p_j}, \tilde{k})$ *for all* $\tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$, $\tilde{k} \tilde{>} \bar{0}$,
(SfM4) $\mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{t}_{p_k}, \tilde{k} \oplus \tilde{l}) \tilde{\geq} \mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) \tilde{\circledast} \mathfrak{S}_F(\tilde{s}_{p_j}, \tilde{t}_{p_k}, \tilde{l})$ *for all* $\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{t}_{p_k} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$, $\tilde{k}, \tilde{l} \tilde{>} \bar{0}$,
(SfM5) $\mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, .) : (0,\infty)(\mathcal{P}) \longrightarrow [0,1](\mathcal{P})$ *is a continuous map.*

A soft fuzzy metric $\mathfrak{S}_F$ together with the absolute soft set $\tilde{\mathcal{K}}_{\mathcal{P}}$ is known as a soft fuzzy metric space. It is denoted as $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ and abbreviated as SFMS.

Example 2. *We consider an SMS $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{L})$. We let $\tilde{\lambda} \tilde{\circledast} \tilde{\mu} = min\{\tilde{\lambda}, \tilde{\mu}\}$ or $\tilde{\lambda} \tilde{\circledast} \tilde{\mu} = \tilde{\lambda} \circ \tilde{\mu}$ be defined in $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{L})$. We define mapping $\mathfrak{S}_F : \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times (0,\infty)(\mathcal{P}) \longrightarrow [0,1](\mathcal{P})$ as*

$$\mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) = \frac{\tilde{k}}{\tilde{k} \oplus \mathfrak{L}(\tilde{r}_{p_i}, \tilde{s}_{p_j})},$$

where $\tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$ and $\tilde{k} \tilde{>} \bar{0}$.

Then, $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ is an SFMS. Moreover, the soft fuzzy metric $\mathfrak{S}_F$ induced by the soft metric $\mathfrak{L}$ is known as a standard soft fuzzy metric.

Definition 13. *We consider* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *as an SFMS. Collection of soft sets* $\Omega = \{(\mathcal{M}, \mathcal{P}) : (\mathcal{M}, \mathcal{P}) \tilde{\subset} (\mathcal{K}, \mathcal{P})\}$ *is said to be a soft open cover of* $\tilde{\mathcal{K}}_{\mathcal{P}}$ *if each* $(\mathcal{M}, \mathcal{P}) \in \Omega$ *is soft open and* $\tilde{\mathcal{K}}_{\mathcal{P}} \tilde{\subset} \tilde{\bigcup}_{(\mathcal{M},\mathcal{P})\in\Omega} (\mathcal{M}, \mathcal{P})$ *[30].*

An SFMS $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *is purported as a compact SFMS if, to each soft open cover of* $\tilde{\mathcal{K}}_{\mathcal{P}}$ *in* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$*, there is a finite assembly of soft open sets* $\{(\mathcal{M}_1, \mathcal{P}_1), (\mathcal{M}_2, \mathcal{P}_2), \ldots, (\mathcal{M}_n, \mathcal{P}_n)\}$ *where* $(\mathcal{M}_i, \mathcal{P}_i) \in \Omega$ *for all* $i \in \{1, 2, \ldots, n\}$ *satisfying* $\tilde{\mathcal{K}}_{\mathcal{P}} \tilde{\subset} \tilde{\bigcup}_{i=1}^{n} (\mathcal{M}_i, \mathcal{P}_i)$*.*

Definition 14. *Any soft sequence* $\{\tilde{r}^m_{p_i}\}$ *in SFMS* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *is said to be convergent to a soft point* $\tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$ *if [6]*

$$\lim_{m\to\infty} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{s}_{p_j}, \tilde{k}) = \bar{1}, \quad \forall\, \tilde{k} \tilde{>} \bar{0}.$$

Equivalently, for any given $\tilde{\varepsilon} \tilde{\in} (\bar{0}, \bar{1})$ *and* $\tilde{k} \tilde{>} \bar{0}$*, there exists* $N_0 \in \mathbb{Z}^+$ *such that*

$$\tilde{r}^m_{p_i} \in SS(\mathcal{B}_{\mathfrak{S}_F}(\tilde{s}_{p_j}, \tilde{\varepsilon}, \tilde{k})), \quad \forall\, m \geq N_0,$$

where $\mathcal{B}_{\mathfrak{S}_F}(\tilde{s}_{p_j}, \tilde{\varepsilon}, \tilde{k})$ *is a soft open ball centred at* $\tilde{s}_{p_j}$ *with radius* $\tilde{\varepsilon}$ *w.r.t.* $\tilde{k}$*. This means*

$$\mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{s}_{p_j}, \tilde{k}) \tilde{>} \bar{1} \ominus \tilde{\varepsilon}, \quad \forall\, \tilde{k} \tilde{>} \bar{0},\ m \geq N_0.$$

Definition 15. *Any soft sequence* $\{\tilde{r}^m_{p_i}\}$ *in SFMS* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *is purported to be a Cauchy sequence in SFMS if [6]*

$$\lim_{m\to\infty} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^t_{p_i}, \tilde{k}) = \bar{1}, \quad \forall\, \tilde{k} \tilde{>} \bar{0}.$$

Equivalently, for any given $\tilde{\varepsilon} \tilde{\in} (\bar{0}, \bar{1})$ *and* $\tilde{k} \tilde{>} \bar{0}$*, there exists* $N_0 \in \mathbb{Z}^+$ *such that*

$$\mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^t_{p_i}, \tilde{k}) \tilde{>} \bar{1} \ominus \tilde{\varepsilon}, \quad \forall\ m, t \geq N_0.$$

Definition 16. *An SFMS* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *is complete if all Cauchy sequences in the SFMS turn out to be convergent [6].*

Definition 17. *An SFMS* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *is compact if all the soft fuzzy sequences in* $\tilde{\mathcal{K}}_{\mathcal{P}}$ *admit at least one convergent soft subsequence [6].*

3. Main Results

Definition 18. *We consider* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *as an SFMS. Soft mapping* $(\mathcal{U}, \Phi) : (\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast}) \to (\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *is purported to be a soft fuzzy contraction if there exists an* $\bar{\alpha} \tilde{\in} [\bar{0}, \bar{1})$ *satisfying the condition:*

$$\mathfrak{S}_F((\mathcal{U}, \Phi)\tilde{r}_{p_i}, (\mathcal{U}, \Phi)\tilde{s}_{p_j}, \tilde{k}) \tilde{\geq} \mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \frac{\tilde{k}}{\bar{\alpha}}) \quad \forall\, \tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}} \text{ and } \tilde{k} \tilde{>} \bar{0}.$$

Definition 19. *Map* $\psi : \mathbb{R}(\mathcal{P}) \longrightarrow [0, \infty)(\mathcal{P})$ *is said to be a* Ψ*- function if it follows the conditions below:*

(i) $\psi(\tilde{k}) = \bar{0} \iff \tilde{k} = \bar{0}$,
(ii) ψ *is increasing ,* $\psi(\tilde{k}) \to \infty_s$ *with* $\tilde{k} \to \infty$,
(iii) *At* $\tilde{k} \tilde{>} \bar{0}$, ψ *is left continuous,*
(iv) *At* $\tilde{k} = \bar{0}$, ψ *is continuous.*

Example 3. *We assume* $\mathbb{R}(\mathcal{P})$ *as a collection of all soft real numbers with soft topology and* $[0, \infty)(\mathcal{P})$ *as the non-negative portion of* $\mathbb{R}(\mathcal{P})$*. We define function* $\psi : \mathbb{R}(\mathcal{P}) \longrightarrow [0, \infty)(\mathcal{P})$ *as follows:*

$$\psi(\tilde{k}) = \begin{cases} \bar{0} & \text{if} \quad \tilde{k} = \bar{0} \\ \sqrt[3]{k} & \text{if} \quad \tilde{k} \neq \bar{0}. \end{cases}$$

Then, ψ *holds all the conditions for a* Ψ*-function.*

Definition 20. *We consider* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *as an SFMS. Soft mapping* $(\mathcal{U}, \Phi) : (\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast}) \to (\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *is said to be a* Ψ*-contraction mapping on SFMS if there exists a soft real number* $\bar{\alpha} \tilde{\in} [\bar{0}, \bar{1})$ *satisfying the condition:*

$$\mathfrak{S}_F((\mathcal{U},\Phi)\tilde{r}_{p_i}, (\mathcal{U},\Phi)\tilde{s}_{p_j}, \psi(\tilde{k})) \tilde{\geq} \mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \psi(\frac{\tilde{k}}{\bar{\alpha}})), \quad \forall\, \tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}} \text{ and } \tilde{k} \tilde{>} \bar{0},$$

where ψ *is a* Ψ*-function.*

Theorem 1. *We consider* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *as a complete SFMS wherein*

$$\lim_{\tilde{k} \to +\infty_s} \mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) = \bar{1} \text{ for all } \tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}. \tag{1}$$

Then, the soft fuzzy contraction mapping $(\mathcal{U}, \Phi)$ *on* $\tilde{\mathcal{K}}_{\mathcal{P}}$ *admits a unique soft fixed point.*

Proof. We consider a soft point $\tilde{r}^0_{p_i} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$ and construct a soft sequence $\{\tilde{r}^m_{p_i}\}$ where $\tilde{r}^m_{p_i} = (\mathcal{U}, \Phi)^m\, \tilde{r}^0_{p_i}$.

Through the induction process, we obtain

$$\mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^{m+1}_{p_i}, \tilde{k}) \tilde{\geq} \mathfrak{S}_F(\tilde{r}^0_{p_i}, \tilde{r}^1_{p_i}, \tilde{k}/\bar{\alpha}^m). \tag{2}$$

Now, by conditions (2) and (SfM4), for any $c \in \mathbb{Z}^+$, we have

$$\begin{aligned} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^{m+c}_{p_i}, \tilde{k}) &\tilde{\geq} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^{m+1}_{p_i}, \tilde{k}/\bar{c}) \underbrace{\tilde{\circledast} \ldots \tilde{\circledast}}_{c-times} \mathfrak{S}_F(\tilde{r}^{m+c-1}_{p_i}, \tilde{r}^{m+c}_{p_i}, \tilde{k}/\bar{c}) \\ &\tilde{\geq} \mathfrak{S}_F(\tilde{r}^0_{p_i}, \tilde{r}^1_{p_i}, \tilde{k}/\bar{c}\bar{\alpha}^m) \underbrace{\tilde{\circledast} \ldots \tilde{\circledast}}_{c-times} \mathfrak{S}_F(\tilde{r}^0_{p_i}, \tilde{r}^1_{p_i}, \tilde{k}/\bar{c}\bar{\alpha}^{m+c-1}). \end{aligned}$$

Now, using (1), we obtain

$$\lim_{\tilde{k} \to +\infty_s} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^{m+c}_{p_i}, \tilde{k}) \tilde{\geq} \underbrace{\bar{1} \tilde{\circledast} \bar{1} \tilde{\circledast} \ldots \tilde{\circledast} \bar{1}}_{c-times} = \bar{1}.$$

Thus, the soft fuzzy sequence $\{\tilde{r}^m_{p_i}\}$ is Cauchy in $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ and hence it is convergent as $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ is complete. We let $\{\tilde{r}^m_{p_i}\} \to \tilde{s}_{p_j}$, $\tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$, i.e.,

$$\lim_{m \to \infty} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{s}^t_{p_j}, \tilde{k}) = \bar{1}. \tag{3}$$

Then,

$$\begin{aligned} \mathfrak{S}_F((\mathcal{U},\Phi)\tilde{s}_{p_j}, \tilde{s}_{p_j}, \tilde{k}) &\tilde{\geq} \mathfrak{S}_F((\mathcal{U},\Phi)\tilde{s}_{p_j}, (\mathcal{U},\Phi)\tilde{r}^m_{p_i}, \tilde{k}/\bar{2}) \tilde{\circledast} \mathfrak{S}_F((\mathcal{U},\Phi)\tilde{r}^m_{p_i}, \tilde{s}_{p_j}, \tilde{k}/\bar{2}). \\ &\tilde{\geq} \mathfrak{S}_F(\tilde{s}_{p_j}, \tilde{r}^m_{p_i}, \tilde{k}/\bar{2}\bar{\alpha}) \tilde{\circledast} \mathfrak{S}_F(\tilde{r}^{m+1}_{p_i}, \tilde{s}_{p_j}, \tilde{k}/\bar{2}) \end{aligned}$$

From (3), we obtain

$$\lim_{m \to \infty} \mathfrak{S}_F((\mathcal{U},\Phi)\tilde{s}_{p_j}, \tilde{s}_{p_j}, \tilde{k}) \tilde{\geq} \bar{1} \tilde{\circledast} \bar{1} = \bar{1},$$

or

$$\lim_{m \to \infty} \mathfrak{S}_F((\mathcal{U},\Phi)\tilde{s}_{p_j}, \tilde{s}_{p_j}, \tilde{k}) = \bar{1}.$$

Hence, $(\mathcal{U}, \Phi)\tilde{s}_{p_j} = \tilde{s}_{p_j}$. Thus, $\tilde{s}_{p_j}$ is a soft fixed point of $(\mathcal{U}, \Phi)$.

The uniqueness of a soft fixed point of the soft fuzzy contraction mapping $(\mathcal{U}, \Phi)$ can be easily verified. □

Theorem 2. *We consider* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *as a complete SFMS with a continuous soft t-norm* $\tilde{\circledast}$ *wherein*

$$\lim_{\tilde{k}\to+\infty_s} \mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) = \bar{1} \;\forall\; \tilde{k}\tilde{>}\bar{0}. \tag{4}$$

Also, we consider a Ψ*-contraction* $(\mathcal{U}, \Phi) : (\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast}) \to (\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$*. Then,* $(\mathcal{U}, \Phi)$ *has a unique soft fixed point.*

Proof. We consider a soft point $\tilde{r}^0_{p_i} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$ and construct a soft sequence $\{\tilde{r}^m_{p_i}\}$ where $\tilde{r}^m_{p_i} = (\mathcal{U}, \Phi)^m\, \tilde{r}^0_{p_i}$.

In accordance with conditions i) and iv) given in Definition 19, for any $\tilde{k}\tilde{>}\bar{0}$, there exists $\tilde{l}\tilde{>}\bar{0}$ such that $\tilde{k}\tilde{>}\psi(\tilde{l})$.

Now, by induction process, we obtain

$$\mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^{m+1}_{p_i}, \tilde{k}) \tilde{\geq} \mathfrak{S}_F(\tilde{r}^0_{p_i}, \tilde{r}^1_{p_i}, \psi(\tilde{l}/\tilde{\alpha}^m)). \tag{5}$$

By utilising conditions (5) and (SfM4), we have, for any $c \in \mathbb{Z}^+$,

$$\begin{aligned}\mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^{m+c}_{p_i}, \tilde{k}) &\tilde{\geq} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^{m+c}_{p_i}, \psi(\tilde{l}))\\ &\tilde{\geq} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^{m+1}_{p_i}, \psi(\tilde{l}/\bar{c}))\ \underbrace{\tilde{\circledast} \ldots \tilde{\circledast}}_{c-times}\ \mathfrak{S}_F(\tilde{r}^{m+c-1}_{p_i}, \tilde{r}^{m+c}_{p_i}, \psi(\tilde{l}/\bar{c}))\\ &\tilde{\geq} \mathfrak{S}_F(\tilde{r}^0_{p_i}, \tilde{r}^1_{p_i}, \psi(\tilde{l}/\bar{c}\tilde{\alpha}^m))\ \underbrace{\tilde{\circledast} \ldots \tilde{\circledast}}_{c-times}\ \mathfrak{S}_F(\tilde{r}^0_{p_i}, \tilde{r}^1_{p_i}, \psi(\tilde{l}/\bar{c}\tilde{\alpha}^{m+c-1})).\end{aligned}$$

Now, letting $\tilde{k} \to +\infty_s$ and using (4), we obtain

$$\lim_{\tilde{k}\to+\infty_s} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{r}^{m+c}_{p_i}, \tilde{k}) \tilde{\geq} \underbrace{\bar{1}\,\tilde{\circledast}\,\bar{1}\,\tilde{\circledast} \ldots \tilde{\circledast}\,\bar{1}}_{c-times} = \bar{1}.$$

Thus, the soft fuzzy sequence $\{\tilde{r}^m_{p_i}\}$ is Cauchy in $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ and hence it is convergent as $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ is complete. We let $\{\tilde{r}^m_{p_i}\} \to \tilde{s}_{p_j}$, $\tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$, i.e.,

$$\lim_{m\to\infty} \mathfrak{S}_F(\tilde{r}^m_{p_i}, \tilde{s}^t_{p_j}, \tilde{k}) = \bar{1}. \tag{6}$$

Also,

$$\begin{aligned}\mathfrak{S}_F((\mathcal{U}, \Phi)\tilde{s}_{p_j}, \tilde{s}_{p_j}, \tilde{k}) &\tilde{\geq} \mathfrak{S}_F((\mathcal{U}, \Phi)\tilde{s}_{p_j}, (\mathcal{U}, \Phi)\tilde{r}^m_{p_i}, \tilde{k}/\bar{2})\ \tilde{\circledast}\ \mathfrak{S}_F((\mathcal{U}, \Phi)\tilde{r}^m_{p_i}, \tilde{s}_{p_j}, \tilde{k}/\bar{2})\\ &\tilde{\geq} \mathfrak{S}_F(\tilde{s}_{p_j}, \tilde{r}^m_{p_i}, \psi(\tilde{l}/\bar{2}\tilde{\alpha}))\ \tilde{\circledast}\ \mathfrak{S}_F(\tilde{r}^{m+1}_{p_i}, \tilde{s}_{p_j}, \tilde{k}/\bar{2}).\end{aligned}$$

From (6) and the fact that $\tilde{\circledast}$ is a continuous soft t-norm, we obtain

$$\mathfrak{S}_F((\mathcal{U}, \Phi)\tilde{s}_{p_j}, \tilde{s}_{p_j}, \tilde{k}) \longrightarrow \bar{1} \text{ as } m \to \infty.$$

Thus, $\tilde{s}_{p_j}$ is a soft fixed point of $(\mathcal{U}, \Phi)$.

The uniqueness of a soft fixed point of the Ψ-contraction function $(\mathcal{U}, \Phi)$ on $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ can be easily proved. □

Theorem 3. *We consider* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *as a complete SFMS with a continuous soft t-norm* $\tilde{\circledast}$ *and* $(\mathcal{U}, \Phi) : (\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast}) \to (\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *as a* Ψ*-contraction mapping. In addition, we assume that for a soft point* $\tilde{r}^0_{p_i} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$*, the iterated soft sequence* $\{\tilde{r}^m_{p_i}\}$ *formed as* $\tilde{r}^m_{p_i} = (\mathcal{U}, \Phi)\tilde{r}^{m-1}_{p_i}, m = 1, 2, 3, \ldots$ *is convergent. Then, a unique soft fixed point of* $(\mathcal{U}, \Phi)$ *exists in* $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *to which* $\{\tilde{r}^m_{p_i}\}$ *converges.*

Proof. We consider a Ψ-contraction mapping $(\mathcal{U},\Phi)$ on $(\tilde{\mathcal{K}}_{\mathcal{P}},\mathfrak{S}_F,\tilde{\circledast})$. Then, there exists a soft real number $\bar{\alpha}\tilde{\in}[\bar{0},\bar{1})$ satisfying the condition

$$\mathfrak{S}_F((\mathcal{U},\Phi)\tilde{r}_{p_i},(\mathcal{U},\Phi)\tilde{s}_{p_j},\psi(\tilde{k}))\tilde{\geq}\mathfrak{S}_F\left(\tilde{r}_{p_i},\tilde{s}_{p_j},\psi\left(\frac{\tilde{k}}{\bar{\alpha}}\right)\right)\quad\forall\,\tilde{r}_{p_i},\tilde{s}_{p_j}\tilde{\in}\tilde{\mathcal{K}}_{\mathcal{P}},\quad\tilde{k}\tilde{>}\bar{0},$$

where ψ is a Ψ-function.

In accordance with requirements (i) and (iv) given in Definition 19, to any $\tilde{k}\tilde{>}\bar{0}$, there exists $\tilde{l}\tilde{>}\bar{0}$ such that $\tilde{k}\tilde{>}\psi(\tilde{l})$.

Therefore, we deduce

$$\mathfrak{S}_F(\tilde{r}^m_{p_i},\tilde{r}^{m+1}_{p_i},\tilde{k})\tilde{\geq}\mathfrak{S}_F(\tilde{r}^0_{p_i},\tilde{r}^1_{p_i},\psi(\tilde{l}/\bar{\alpha}^m)).\tag{7}$$

We let $m\to\infty$ in condition (7). Then, $\mathfrak{S}_F(\tilde{r}^m_{p_i},\tilde{r}^{m+1}_{p_i},\tilde{k})\longrightarrow\bar{1}$.

Now, since $\{\tilde{r}^m_{p_i}\}$ is convergent, there is a soft point $\tilde{s}_{p_j}\tilde{\in}\tilde{\mathcal{K}}_{\mathcal{P}}$ such that $\{\tilde{r}^m_{p_i}\}\to\tilde{s}_{p_j}$, i.e.,

$$\mathfrak{S}_F(\tilde{r}^m_{p_i},\tilde{s}^t_{p_j},\tilde{k})\longrightarrow\bar{1}\text{ as }m\to\infty.\tag{8}$$

Thus,

$$\begin{aligned}\mathfrak{S}_F((\mathcal{U},\Phi)\tilde{s}_{p_j},\tilde{s}_{p_j},\tilde{k})&\tilde{\geq}\mathfrak{S}_F((\mathcal{U},\Phi)\tilde{s}_{p_j},(\mathcal{U},\Phi)\tilde{r}^m_{p_i},\tilde{k}/\bar{2})\,\tilde{\circledast}\,\mathfrak{S}_F((\mathcal{U},\Phi)\tilde{r}^m_{p_i},\tilde{s}_{p_j},\tilde{k}/\bar{2}).\\&\tilde{\geq}\mathfrak{S}_F(\tilde{s}_{p_j},\tilde{r}^m_{p_i},\psi(\tilde{l}/\bar{2}\bar{\alpha}))\,\tilde{\circledast}\,\mathfrak{S}_F(\tilde{r}^{m+1}_{p_i},\tilde{s}_{p_j},\tilde{k}/\bar{2})\end{aligned}$$

From (8) and the fact that $\tilde{\circledast}$ is a continuous soft t-norm,

$$\mathfrak{S}_F((\mathcal{U},\Phi)\tilde{s}_{p_j},\tilde{s}_{p_j},\tilde{k})\longrightarrow\bar{1}\text{ as }m\to\infty.$$

Thus, $\tilde{s}_{p_j}$ is a fixed point of $(\mathcal{U},\Phi)$.

Ultimately, the uniqueness of a soft fixed point of the Ψ-contraction map $(\mathcal{U},\Phi)$ on $(\tilde{\mathcal{K}}_{\mathcal{P}},\mathfrak{S}_F,\tilde{\circledast})$ can be easily verified. □

Theorem 4. *We consider $(\tilde{\mathcal{K}}_{\mathcal{P}},\mathfrak{S}_F,\tilde{\circledast})$ as a complete SFMS with a continuous soft t-norm $\tilde{\circledast}$ described as $\tilde{\lambda}\tilde{\circledast}\tilde{\mu}=min\{\tilde{\lambda},\tilde{\mu}\}$. Also, we consider a Ψ-contraction $(\mathcal{U},\Phi):(\tilde{\mathcal{K}}_{\mathcal{P}},\mathfrak{S}_F,\tilde{\circledast})\to(\tilde{\mathcal{K}}_{\mathcal{P}},\mathfrak{S}_F,\tilde{\circledast})$. Then, $(\mathcal{U},\Phi)$ has a unique soft fixed point.*

Proof. We consider a soft point $\tilde{r}^0_{p_i}\tilde{\in}\tilde{\mathcal{K}}_{\mathcal{P}}$. Form soft sequence $\{\tilde{r}^m_{p_i}\}$ as below,

$$\tilde{r}^m_{p_i}=(\mathcal{U},\Phi)^m\tilde{r}^0_{p_i}.$$

In line with Theorem 3, the proof is complete, reaffirming that $\{\tilde{r}^m_{p_i}\}$ is a Cauchy soft sequence.

We assume $\{\tilde{r}^m_{p_i}\}$ is not a Cauchy soft sequence. Then, there exist soft real numbers $\tilde{k}\tilde{>}\bar{0}$ and $\tilde{\varepsilon}\tilde{>}\bar{0}$ satisfying that, for any $N_0\in\mathbb{Z}^+$, there exists $m(N_0),t(N_0)\geq N_0$ such that

$$\mathfrak{S}_F(\tilde{r}^{m(N_0)}_{p_i},\tilde{r}^{t(N_0)}_{p_i},\tilde{k})\tilde{>}\bar{1}\ominus\tilde{\varepsilon},\tag{9}$$

choosing $m(N_0)<t(N_0)$ so that $t(N_0)$ is the lowest positive integer with respect to $m(N_0)$ which satisfies condition (9).

Then, there exists $\tilde{k}\tilde{>}\bar{0}$ and $\tilde{\varepsilon}\tilde{>}\bar{0}$ for which two increasing sequences $\{t(N_0)\}$ and $\{m(N_0)\}$, $t(N_0)>m(N_0)$ can be formed, which satisfies the following:

$$\mathfrak{S}_F(\tilde{r}^{m(N_0)}_{p_i},\tilde{r}^{t(N_0)-1}_{p_i},\tilde{k})\tilde{\leq}\bar{1}\ominus\tilde{\varepsilon}\tag{10}$$

and

$$\mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{r}_{p_i}^{t(N_0)}, \tilde{k}) \tilde{>} \bar{1} \ominus \tilde{\varepsilon}. \tag{11}$$

For the formation of such sequences, it is required to find a soft point $\tilde{r}_{p_i}^{t(N_0)}$ such that

$$\tilde{r}_{p_i}^{t(N_0)} \tilde{\notin} \{\tilde{s}_{p_j} : \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{s}_{p_j}, \tilde{k}) \tilde{\leq} \bar{1} \ominus \tilde{\varepsilon}\} \text{ and } \tilde{r}_{p_i}^{t(N_0)-1} \tilde{\in} \{\tilde{s}_{p_j} : \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{s}_{p_j}, \tilde{k}) \tilde{\leq} \bar{1} \ominus \tilde{\varepsilon}\}.$$

Construction of such a sequence is possible as it is assumed that $\{\tilde{r}_{p_i}^{m}\}$ is not a Cauchy soft sequence.

Since for $\tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}, \tilde{\varepsilon} \tilde{>} \bar{0}$ and $\bar{0} \tilde{<} \tilde{k_1} \tilde{<} \tilde{k_2}$,

$$\{\tilde{s}_{p_j} : \mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k_1}) \tilde{\leq} \bar{1} \ominus \tilde{\varepsilon}\} \tilde{\subset} \{\tilde{s}_{p_j} : \mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k_2}) \tilde{\leq} \bar{1} \ominus \tilde{\varepsilon}\},$$

it follows that whenever such sequence formation is attainable for $\tilde{k} \tilde{>} \bar{0}, \tilde{\varepsilon} \tilde{>} \bar{0}$, the construction of $\{\tilde{r}_{p_i}^{m(N_0)}\}$ and $\{\tilde{r}_{p_i}^{t(N_0)}\}$ satisfies Conditions (10) and (11) corresponding to any $\tilde{l} \tilde{>} \bar{0}, \tilde{\varepsilon} \tilde{>} \bar{0}$ where $\tilde{l} \tilde{<} \tilde{k}$.

Now, as ψ is a Ψ-function, for any $\tilde{k} \tilde{>} \bar{0}$, there exists $\tilde{l} \tilde{>} \bar{0}$ such that $\tilde{k} \tilde{>} \psi(\tilde{l})$. Therefore, we take $\tilde{k}$ in (10) and (11) as $\tilde{k} = \psi(\tilde{k_1})$ for some $\tilde{k_1} \tilde{>} \bar{0}$ such that $\psi(\tilde{k_1}/\bar{\alpha}) \tilde{>} \psi(\tilde{k_1})$. Such a choice is possible through requirements i) and iv) given in Definition 19.

By the Conditions (10) and (11), we obtain

$$\mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{r}_{p_i}^{t(N_0)-1}, \psi(\tilde{k_1})) \tilde{\leq} \bar{1} \ominus \tilde{\varepsilon} \tag{12}$$

and

$$\mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{r}_{p_i}^{t(N_0)}, \psi(\tilde{k_1})) \tilde{>} \bar{1} \ominus \tilde{\varepsilon}. \tag{13}$$

Thus,

$$\begin{aligned} \bar{1} \ominus \tilde{\varepsilon} &\tilde{<} \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{r}_{p_i}^{t(N_0)}, \psi(\tilde{k_1})) \\ &\tilde{\leq} \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)-1}, \tilde{r}_{p_i}^{t(N_0)-1}, \psi(\tilde{k_1}/\bar{\alpha})), \end{aligned}$$

or

$$\bar{1} \ominus \tilde{\varepsilon} \tilde{<} \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)-1}, \tilde{r}_{p_i}^{t(N_0)-1}, \psi(\tilde{k_1}/\bar{\alpha})).$$

As $\psi(\tilde{k_1}/\bar{\alpha}) \tilde{>} \psi(\tilde{k_1})$, choosing $\tilde{K}$ as $\tilde{K} \tilde{<} \{\psi(\tilde{k_1}/\bar{\alpha}) \ominus \psi(\tilde{k_1})\}$.

This means $\psi(\tilde{k_1}/\bar{\alpha}) \ominus \tilde{K} \tilde{>} \psi(\tilde{k_1})$.

Through Condition (7) in Theorem 3, we choose N_0 large enough such that

$$\mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{r}_{p_i}^{m(N_0)-1}, \tilde{K}) \tilde{<} \bar{1} \ominus \tilde{\varepsilon}_1 \text{ for } \bar{0} \tilde{<} \tilde{\varepsilon}_1 \tilde{<} \tilde{\varepsilon}. \tag{14}$$

With this choice of N_0 and $\tilde{K}$ and by Conditions (12)–(14) we obtain

$$\begin{aligned} \bar{1} \ominus \tilde{\varepsilon} &\tilde{<} \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{r}_{p_i}^{t(N_0)-1}, \psi(\tilde{k_1}/\bar{\alpha})) \\ &\tilde{\leq} \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{r}_{p_i}^{t(N_0)-1}, (\psi(\tilde{k_1}/\bar{\alpha}) \ominus \tilde{K})) \tilde{\circledast} \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)-1}, \tilde{r}_{p_i}^{m(N_0)}, \tilde{K}) \\ &\tilde{\leq} \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)}, \tilde{r}_{p_i}^{t(N_0)-1}, \psi(\tilde{k_1})) \tilde{\circledast} \mathfrak{S}_F(\tilde{r}_{p_i}^{m(N_0)-1}, \tilde{r}_{p_i}^{m(N_0)}, \tilde{K}) \\ &\tilde{\leq} (\bar{1} \ominus \tilde{\varepsilon}) \tilde{\circledast} (\bar{1} \ominus \tilde{\varepsilon}_1), \end{aligned}$$

and using the fact that $\tilde{\varepsilon}_1 \tilde{<} \tilde{\varepsilon}$, we have $(\bar{1} \ominus \tilde{\varepsilon}) \tilde{<} (\bar{1} \ominus \tilde{\varepsilon}_1)$.

This introduces a contradiction. As a result, $\{\tilde{r}_{p_i}^{m}\}$ is Cauchy. The proof follows Theorem 3 after that. □

4. Illustrations

In this section, we include some numerical illustrations that reinforce the established theorems proved in Section 3. The soft fuzzy Banach contraction theorem described in Theorem 1 is confirmed by Examples 4 and 5, and Example 6 supports Theorem 4.

Example 4. *We consider a set $\mathcal{K} = \{r, s, t\}$ and a parameter set $\mathcal{P} = \{a, b\}$ with a soft t-norm defined as $\tilde{c}\tilde{\circledast}\tilde{d} = min\{\tilde{c}, \tilde{d}\}$. Then, $\mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) = \{\tilde{r}_a, \tilde{r}_b, \tilde{s}_a, \tilde{s}_b, \tilde{t}_a, \tilde{t}_b\}$.*

We define $\mathfrak{S}_F : \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times (0,\infty)(\mathcal{P}) \longrightarrow [0,1](\mathcal{P})$ as follows:
for any $p_i, p_j \in \mathcal{P}$,

$$\mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) = \mathfrak{S}_F(\tilde{s}_{p_j}, \tilde{r}_{p_i}, \tilde{k}) = \begin{cases} \bar{0} & if \quad \tilde{k} = \bar{0} \\ \overline{0.8} & if \quad \bar{0}\tilde{<}\tilde{k}\tilde{\leq}\bar{2} \\ \bar{1} & if \quad \tilde{k}\tilde{>}\bar{2}, \end{cases}$$

$$\mathfrak{S}_F(\tilde{t}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) = \mathfrak{S}_F(\tilde{s}_{p_j}, \tilde{t}_{p_i}, \tilde{k}) = \mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{t}_{p_j}, \tilde{k}) = \mathfrak{S}_F(\tilde{t}_{p_j}, \tilde{r}_{p_i}, \tilde{k}) = \begin{cases} \bar{0} & if \quad \tilde{k} = \bar{0} \\ \overline{0.5} & if \quad \bar{0}\tilde{<}\tilde{k}\tilde{\leq}\bar{4} \\ \bar{1} & if \quad \tilde{k}\tilde{>}\bar{4}, \end{cases}$$

$\mathfrak{S}_F(\tilde{r}_{p_i}, \tilde{s}_{p_j}, \tilde{k}) = \bar{1} \iff \tilde{r}_{p_i} = \tilde{s}_{p_j}$ for all $\tilde{r}_{p_i}, \tilde{s}_{p_j} \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$ and $\tilde{k}\tilde{>}\bar{0}$.

Then, $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ is a complete SFMS.

Now, we consider a soft self-mapping $(\mathcal{U}, \Phi)$ on $\tilde{\mathcal{K}}_{\mathcal{P}}$ defined as
$(\mathcal{U}, \Phi)(\tilde{r}_a) = \tilde{s}_a, (\mathcal{U}, \Phi)(\tilde{r}_b) = \tilde{s}_b, (\mathcal{U}, \Phi)(\tilde{s}_a) = \tilde{s}_b,$
$(\mathcal{U}, \Phi)(\tilde{s}_b) = \tilde{s}_b, (\mathcal{U}, \Phi)(\tilde{t}_a) = \tilde{r}_b, (\mathcal{U}, \Phi)(\tilde{t}_b) = \tilde{r}_a.$

Then, $(\mathcal{U}, \Phi)$ is a soft contraction map on SFMS $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ and it follows all the conditions specified in Theorem 1. Moreover, it admits only one fixed point, which is $\tilde{s}_b$.

Example 5. *We consider a set $\mathcal{K} = U \cup V$, where $U = \{\frac{1}{2}, \frac{1}{3}\}$, $V = [4,5]$, and a parameter set $P = \{1,2\}$. We describe $\mathfrak{L} : SP(\tilde{\mathcal{K}}_P) \times SP(\tilde{\mathcal{K}}_P) \rightarrow \mathbb{R}(P)^*$ as follows: $\mathfrak{L}(\tilde{r}_p, \tilde{s}_q) = |\bar{r} - \bar{s}| + |\bar{p} - \bar{q}|\ \forall\ \tilde{r}_p, \tilde{s}_q \in SP(\tilde{\mathcal{K}}_P)$.*

In $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{L})$, we define $\tilde{r}\tilde{\circledast}\tilde{s} = \tilde{r} \circ \tilde{s}$ or $\tilde{r}\tilde{\circledast}\tilde{s} = min\{\tilde{r}, \tilde{s}\}$. We define $\mathfrak{S}_F : \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times (0,\infty)(\mathcal{P}) \longrightarrow [0,1](\mathcal{P})$ as follows:

$$\mathfrak{S}_F(\tilde{r}_p, \tilde{s}_q, \tilde{k}) = \frac{\tilde{k}}{\tilde{k} \oplus \mathfrak{L}(\tilde{r}_p, \tilde{s}_q)}$$

for each $\tilde{r}_p, \tilde{s}_q \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}$ and $\tilde{k}\tilde{>}\bar{0}$.

Then, $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ is a complete SFMS.

Now, we consider map $(\mathcal{U}, \Phi) : \tilde{\mathcal{K}}_{\mathcal{P}} \rightarrow \tilde{\mathcal{K}}_{\mathcal{P}}$ defined by

$$(\mathcal{U}, \Phi)(\tilde{r}_p) = \begin{cases} (\tilde{\frac{1}{2}})_1 & if \quad \tilde{r}_p \in SP(\tilde{V}_P) \\ (\tilde{\frac{1}{3}})_2 & otherwise. \end{cases}$$

Then, $(\mathcal{U}, \Phi)$ is a soft contraction map on SFMS $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ and it follows all the conditions specified in Theorem 1. Moreover, it admits only one fixed point, i.e., $(\tilde{\frac{1}{3}})_2$.

Example 6. *We onsider set $\mathcal{K} = \{\frac{5}{8}, \frac{3}{4}, \frac{8}{9}\}$ and parameter set $\mathcal{P} = \{1,2\}$ with a soft t-norm defined as $\tilde{c}\tilde{\circledast}\tilde{d} = min\{\tilde{c}, \tilde{d}\}$ for $\tilde{c}, \tilde{d} \in [0,1](\mathcal{P})$. Then, $\mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) = \{\tilde{\frac{5}{8}}_1, \tilde{\frac{5}{8}}_2, \tilde{\frac{3}{4}}_1, \tilde{\frac{3}{4}}_2, \tilde{\frac{8}{9}}_1, \tilde{\frac{8}{9}}_2\}$. We define $\mathfrak{S}_F : \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times \mathcal{SP}(\tilde{\mathcal{K}}_{\mathcal{P}}) \times (0,\infty)(\mathcal{P}) \longrightarrow [0,1](\mathcal{P})$ as follows:*
for any $p, q \in \mathcal{P}$,

$$\mathfrak{S}_F(\tilde{\frac{5}{8}}_p, \tilde{\frac{3}{4}}_q, \tilde{k}) = \mathfrak{S}_F(\tilde{\frac{3}{4}}_q, \tilde{\frac{5}{8}}_p, \tilde{k}) = \begin{cases} \bar{0} & if \quad \tilde{k} = \bar{0} \\ \overline{0.9} & if \quad \bar{0}\tilde{<}\tilde{k}\tilde{\leq}\bar{3} \\ \bar{1} & if \quad \tilde{k}\tilde{>}\bar{3}, \end{cases}$$

$$\mathfrak{S}_F(\frac{\tilde{8}}{9}_p, \frac{\tilde{3}}{4}_q, \tilde{k}) = \mathfrak{S}_F(\frac{\tilde{3}}{4}_q, \frac{\tilde{8}}{9}_p, \tilde{k}) = \mathfrak{S}_F(\frac{\tilde{5}}{8}_p, \frac{\tilde{8}}{9}_q, \tilde{k}) = \mathfrak{S}_F(\frac{\tilde{8}}{9}_q, \frac{\tilde{5}}{8}_p, \tilde{k}) = \begin{cases} \bar{0} & if \quad \tilde{k} = \bar{0} \\ \overline{0.6} & if \quad \bar{0} \tilde{<} \tilde{k} \tilde{\leq} \bar{8} \\ \bar{1} & if \quad \tilde{k} \tilde{>} \bar{8}, \end{cases}$$

$\mathfrak{S}_F(\tilde{r}_p, \tilde{s}_q, \tilde{k}) = \bar{1} \iff \tilde{r}_p = \tilde{s}_p,$ *for all* $\tilde{r}_p, \tilde{s}_q \tilde{\in} \tilde{\mathcal{K}}_{\mathcal{P}}, \tilde{k} \tilde{>} \bar{0}$.

Then, $(\tilde{\mathcal{K}}_{\mathcal{P}}, \mathfrak{S}_F, \tilde{\circledast})$ *is a complete SFMS.*

Now, we consider a soft self-map $(\mathcal{U}, \Phi)$ *on* $\tilde{\mathcal{K}}_{\mathcal{P}}$ *as*

$(\mathcal{U}, \Phi)(\frac{\tilde{5}}{8}_1) = \frac{\tilde{3}}{4}_1$; $(\mathcal{U}, \Phi)(\frac{\tilde{5}}{8}_2) = \frac{\tilde{3}}{4}_1$;
$(\mathcal{U}, \Phi)(\frac{\tilde{3}}{4}_1) = \frac{\tilde{3}}{4}_1$; $(\mathcal{U}, \Phi)(\frac{\tilde{3}}{4}_2) = \frac{\tilde{5}}{8}_2$;
$(\mathcal{U}, \Phi)(\frac{\tilde{8}}{9}_1) = \frac{\tilde{8}}{9}_2$; $(\mathcal{U}, \Phi)(\frac{\tilde{8}}{9}_2) = \frac{\tilde{8}}{9}_1$.

We let $\psi(\tilde{k}) = \bar{k}^{\frac{1}{3}}$. *Then,* $\psi \in \Psi$. *Here,* $(\mathcal{U}, \Phi)$ *follows the requirements outlined in Theorem 4 and also admits a unique fixed point* $\frac{\tilde{3}}{4}_1$.

5. Conclusions

In this paper, an unfamiliar version of Banach contraction mapping is presented in a soft fuzzy metric space, and a set of fixed point results is achieved by employing a certain restriction on the soft fuzzy metric between the soft points of the absolute soft set taken into consideration. The analysis for broadening the contraction principle complies with the presented work. The concept of altering distance functions is expanded in the context of soft fuzzy metric spaces by introducing Ψ-contraction mapping on soft fuzzy metric spaces. By considering Ψ-contraction mapping and the continuity of the soft t-norm, a set of results for a unique fixed point is also established. Additionally, some appropriate illustrations are provided in support of the established fixed point results. The present work can be applied to real-life problems dealing with uncertainty to obtain a more precise response. The results can be extended to various generalizations of metric spaces, such as neutrosophic soft fuzzy metric spaces, soft fuzzy partial metric spaces, soft fuzzy b metric spaces, and many more. The established results can be extended to the best proximity point results. Moreover, some applications of the established results can be found in theoretical and computational mathematics.

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